

## Assignment 9

Coverage: 16.2, 16.3 in Text.

Exercises: 16.2 no 29, 32, 37, 41, 42, 43. 16.3 no 2, 5, 9, 11, 15, 16, 18, 20, 27, 29.

Hand in 16.2 no 32, 42; 16.3 no 11, 16, 20 by March 22.

### Supplementary Problems

This problem is optional.

1. Let  $F = (F_1, \dots, F_n)$  be a smooth vector field in an open region in  $\mathbb{R}^n$ . Show that if it is conservative, then the component test hold

$$\frac{\partial F_i}{\partial x_j} = \frac{\partial F_j}{\partial x_i}, \quad \forall i, j.$$

2. A region is called star-shaped if there is a point  $O$  inside so that the line segment connecting any point in this region to  $O$  lies completely in this region. For simplicity take  $O$  to be the origin.

- (a) Show that in case the vector field  $\mathbf{F}$  admits a potential  $g$  in this region, then

$$g(x, y, z) = \int_0^1 \mathbf{F}(tx, ty, tz) \cdot (x\mathbf{i} + y\mathbf{j} + z\mathbf{k}) dt .$$

- (b) Show that when  $\mathbf{F}$  passes the component test, the above formula defines a potential function for  $\mathbf{F}$ .